



Mixed convective–magnetohydrodynamic flow of a micropolar fluid with ohmic heating, radiation and viscous dissipation over a chemically reacting porous plate subjected to a constant heat flux and concentration gradient

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(Received 12 June 2012, revised 11 October 2013)

Abstract: In the present paper, chemically reacting mixed convective–magnetohydrodynamic (MHD) micropolar flow, heat and mass transfer in a porous medium with the effects of ohmic heating, radiation and viscous dissipation past an infinite vertical plate, which was subjected to a constant heat flux and concentration gradient, were analyzed. The non-linear coupled partial differential equations were solved numerically using an implicit finite difference scheme known as the Keller-box Method. The results for concentration, transverse velocity, angular velocity and temperature were obtained and illustrated graphically to observe the effects of various parameters, and a numerical discussion is presented with physical interpretations.

Keywords: mixed convection; heat and mass transfer; heat flux; heat generation; ohmic heating; micropolar fluid; chemical reaction.

INTRODUCTION

Flows arising from temperature difference have great significance not only theoretically, but also for applications in geophysics and engineering. There are many interesting aspects of such flows, so analytical solutions of such problems have been presented by many authors, *e.g.*, Gebhart and Pera,¹ Sparrow *et al.*,² Soundalgekar,³ Acharya *et al.*,⁴ Singh and Chand,⁵ *etc.* Investigations of flow streaming into a porous and permeable medium assuming a high velocity of the flow (the Reynolds Number is moderately high) were obtained by Yamamoto and Iwamura,⁶ Yamamoto and Yoshida,⁷ Brinkman⁸ and Raptis *et al.*^{9–10} All the above-mentioned authors used the generalized Darcy Law, and the generalized Darcy Law was derived without taking into account the angular velocity of the fluid particles. Rapti¹¹ in his research paper on a horizontal plate used flow

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doi: 10.2298/JSC130612121K

equations with angular velocity. Raptis¹² in another research paper discussed magnetopolar fluid through a porous medium.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes, In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Diffusion rates could be altered tremendously by chemical reactions. The effect of a chemical reaction depends whether the reaction is homogeneous or heterogeneous. Kandasamy *et al.*¹³ studied thermophoresis and variable viscosity effects on magnetohydrodynamic (MHD) mixed convective heat and mass transfer past a porous wedge in the presence of a chemical reaction. Kandasamy and Devi¹⁴ studied the effects of chemical reaction, and heat and mass transfer on non-linear laminar boundary-layer flow over a wedge with suction or injection. In addition, studies of heat generation or absorption in moving fluids for problems involving chemical reactions and those concerned with dissociating fluids are equally important. Specifically, the effects of heat generation may alter the temperature distribution, consequently affecting the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers. The problem of heat transfer in MHD boundary-layer flow through a porous medium, due to a non-isothermal stretching sheet, with suction, radiation, and heat annihilation was considered by Kumar.¹⁵

Moreover, when the temperature of surrounding fluid is high, radiation effects play an important role and that cannot be ignored,^{16,17} Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such areas of engineering, where high temperature heat transfer occurs. In such cases, the effects of radiation have to be taken into account. Ganesan *et al.*¹⁸ studied the effects of radiation and free convection for an impulsively started infinite vertical isothermal plate using the Rosseland Approximation.¹⁹ The problem of radiative heat transfer with hydro-magnetic flow and viscous dissipation over a stretching surface in the presence of a variable heat flux was solved analytically by Kumar.²⁰ Hossain and Takhar,²¹ Raptis and Massals,²² and Hossain *et al.*²³ studied the radiation effect on free and forced convection flows past a vertical plate, including various physical aspects. Aboeldhab²⁴ studied the radiation effect in heat transfer in an electrically conducting fluid at a stretching surface. At high operating temperature, the radiation effect can be quite significant.²⁵ Heat and mass transfer effects on a moving plate in the presence of thermal radiation were studied by Muthukumarswamy and Kumar²⁶ using the Laplace technique. For the problem of coupled heat and mass transfer in MHD free convection, the effects of both viscous dissipation and ohmic heating were not studied in the above investigations. However, it is more realistic to include these two effects to explore the impact of the magnetic field on the thermal transport in the boundary layer. With this awareness, the effect of

ohmic heating on the MHD free convection heat transfer was examined by Hossain²⁷ for a Newtonian fluid. Chen²⁸ studied the problem of combined heat and mass transfer of an electrically conducting fluid in MHD natural convection, adjacent to a vertical surface with ohmic heating.

In the present work, a study of steady mixed convection flow of a laminar, incompressible, MHD micropolar fluid, with thermal and mass diffusion effects in porous media was performed. The object of the study was to analyze the effects of a magnetic field, the heat source and radiation on the thermal transport in the boundary layer, when the wall was at the prescribed heat flux.

MATHEMATICAL ANALYSIS

Herein, a mixed convection flow of an incompressible and electrically conducting viscous thermo-micropolar fluid past an infinite porous vertical plate is considered. The vertical plate was assumed to be at a constant heat flux and a constant concentration gradient. A magnetic field (B_0) of uniform strength was applied transversely to the direction of the flow, that is the y -axis, and the induced magnetic field was neglected. The x -axis was along the vertical porous plate in the upward direction and the y -axis was normal to it. Since the length of the plate is large and fluid flow extends to infinity, all the physical variables are independent of x and hence functions of y only. The governing equations of continuity, momentum, concentration, angular velocity and energy for the flow in the presence of ohmic heating, radiation, heat generation, chemical reaction and viscous dissipation are:

$$\frac{\partial v^*}{\partial y^*} = 0 \quad (1)$$

$$v^* = -V_0 \text{ (Constant)} \quad (2)$$

$$\frac{dp}{dy^*} = 0 \Rightarrow p \text{ is independent of } y^* \quad (3)$$

$$\begin{aligned} \rho v^* \frac{\partial u^*}{\partial y^*} = & (\kappa + \mu) \frac{\partial^2 u^*}{\partial y^{*2}} + \rho g \beta_T (T - T_\infty) + \rho g \beta_c (c - c_\infty) + \\ & + \kappa \frac{\partial \omega_a}{\partial y^*} - \frac{\mu}{K^*} u^* - \sigma B_0^2 u^* \end{aligned} \quad (4)$$

$$\rho j \left(v \frac{\partial \omega_a}{\partial y^*} \right) = \gamma \frac{\partial^2 \omega_a}{\partial y^{*2}} - 2 \kappa \omega_a \quad (5)$$

$$\rho c_p v^* \frac{\partial T}{\partial y^*} = k \frac{\partial^2 T}{\partial y^{*2}} + \mu \left(\frac{\partial u^*}{\partial y^*} \right)^2 - \frac{\partial q_r}{\partial y^*} + Q(T - T_\infty) + \sigma B_0^2 u^{*2} \quad (6)$$

$$v^* \frac{\partial c}{\partial y^*} = D \frac{\partial^2 c}{\partial y^{*2}} - K_l (c - c_\infty) \quad (7)$$

with the boundary conditions:

$$\begin{aligned} u^* = V_0, \quad \frac{\partial \omega_a}{\partial y^*} = -\frac{\partial^2 u^*}{\partial y^{*2}}, \quad \frac{\partial T}{\partial y^*} = -\frac{q}{k}, \quad -D \frac{\partial c}{\partial y^*} = m_w, \quad \text{at } y = 0 \\ u^* \rightarrow 0, \quad \omega_a \rightarrow 0, \quad T \rightarrow T_\infty, \quad c \rightarrow c_\infty, \quad \text{as } y \rightarrow \infty \end{aligned} \quad (8)$$

where $V_0 > 0$, $\gamma = \left(\mu + \frac{\kappa}{2}\right)j = \mu\left(1 + \frac{a}{2}\right)j$ and $j = \frac{v^2}{V_0^2}$

The Rosseland Approximation²⁰ is assumed for radiative heat flux, which leads to:

$$q_r = -\frac{4\sigma' \partial T^4}{3\kappa^* \partial y^*} \quad (9)$$

If the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature, then the Taylor series for T^4 about T_∞ , after ignoring higher order terms, is given by:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \quad (9a)$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities were introduced:

$$\begin{aligned} y &= \frac{V_0 y^*}{\nu}, \quad u = \frac{u^*}{V_0}, \quad M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, \quad Pr = \frac{\mu c_p}{k}, \quad \theta = \frac{T - T_\infty}{q\nu / k V_0}, \quad C = \frac{c - c_\infty}{m_w \nu / V_0 D}, \\ Ec &= \frac{k V_0^3}{q\nu c_p}, \quad G_r = \frac{g \beta_T q \nu^2}{k V_0^4}, \quad G_c = \frac{g \beta_c m_w \nu^2}{V_0^4 D}, \quad S = \frac{Q\nu}{\rho c_p V_0^2}, \quad N = \frac{\kappa^* k}{4\sigma' T_\infty^3}, \\ \omega &= \frac{\nu \omega_a}{V_0^2}, \quad Sc = \frac{\vartheta}{D}, \quad K_c = \frac{\vartheta K_l}{\nu_w^2}, \quad K = \frac{K^* V_0^2}{\vartheta^2}, \quad a = \frac{\kappa}{\mu} \end{aligned}$$

Equations (4)–(7) change to:

$$(1+a) \frac{d^2 u}{dy^2} + \frac{du}{dy} - \left(M + \frac{1}{K}\right) u + a \frac{d\omega}{dy} + G_r \theta + G_c c = 0 \quad (10)$$

$$\left(1 + \frac{a}{2}\right) \frac{d^2 \omega}{dy^2} + \frac{d\omega}{dy} - 2a\omega = 0 \quad (11)$$

$$\left(1 + \frac{4}{3N}\right) \frac{d^2 \theta}{dy^2} + Pr \frac{d\theta}{dy} + SPr \theta + Pr Ec \left(\frac{du}{dy}\right)^2 + Pr Ec M u^2 = 0 \quad (12)$$

$$\frac{d^2 c}{dy^2} + Sc \frac{dc}{dy} - K_c Sc c = 0 \quad (13)$$

and the boundary conditions change to:

$$\begin{aligned} \text{at } y=0, \quad u=1, \quad \frac{d\theta}{dy} = -1, \quad \frac{d\omega}{dy} = -\frac{d^2 u}{dy^2}, \quad \frac{dc}{dy} = -1 \\ \text{as } y \rightarrow \infty, \quad u \rightarrow 0, \quad \theta \rightarrow 0, \quad \omega \rightarrow 0, \quad c \rightarrow 0 \end{aligned} \quad (14)$$

RESULTS AND NUMERICAL DISCUSSION

Equations (10)–(13), subjected to the boundary conditions (14), were solved numerically using the Keller-box method as described by Cebeci and Bradshaw.^{29,30} The objective of this study was to determine the effects a magnetic field, radiation, viscous dissipation, the Prandtl number and heat source on the fluid

temperature, and the effects of porosity, viscous dissipation and magnetic field on the velocity. The effects of the Schmidt number and chemical reaction on the concentration were determined and the effect of viscous dissipation on the angular velocity was analyzed.

The profiles of concentration for different values of Sc and K_c are presented in Figs. 1 and 2, respectively, which show that the concentration decreases with Sc or K_c . Physically, an increase in Sc means a decrease in molecular diffusion coefficient (D). This results in a decrease in the concentration boundary layer. Hence, the concentration of the species is higher for small values of Sc . Here, the chemical reaction was considered to be homogeneous first-ordered, and as is known, $K_c > 0$ represents a destructive reaction, and $K_c < 0$ a generative reaction. The destructive chemical reaction was taken into account here. Consequently, the concentration decreases for increments of the chemical reaction parameter. In a mixed convection regime, the concentration of the fluid decrease with increasing destructive reaction and thermophoresis particle deposition.

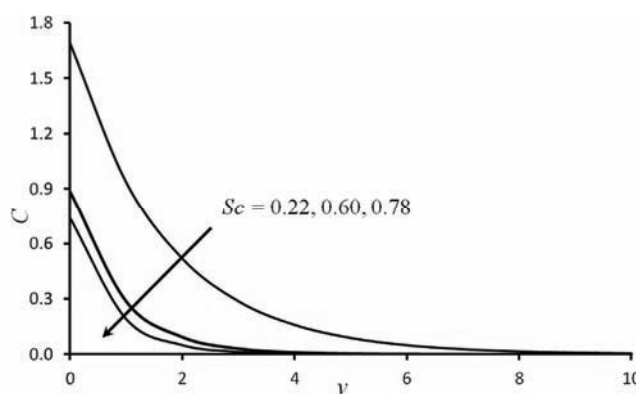


Fig. 1. Concentration for various values of Sc , when $K_c = 1.0$.

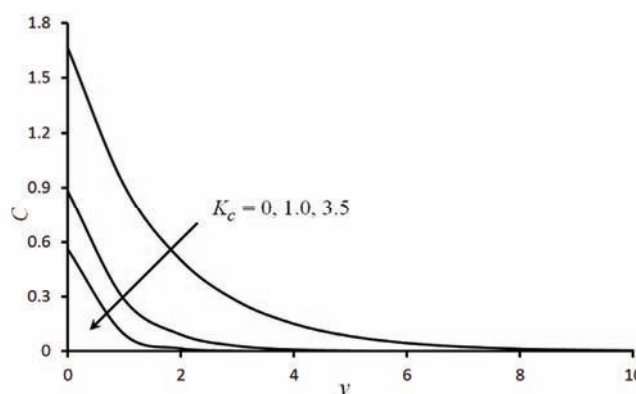


Fig. 2. Concentration for various values of K_c , when $Sc = 0.6$.

The transverse velocity is presented in Figs. 3–6 for different variations in M , K and Ec , respectively. The velocity decreases as M increases, whereas it increases with increasing K or Ec . Physically, the effect of increasing the magnetic field strength is to increase the retarding force and hence reduce the velocity. An increase in the porosity parameter physically means to reduce the drag force and hence causes the flow velocity to increase. An increase in K will reduce the resistance of the porous medium, which leads to an increase in the velocity. The effect of Ec in the flow field is to increase the energy, yielding a greater buoyancy force; this increase in the buoyancy force due to an increase in the dissipation parameter enhances the convective velocity.

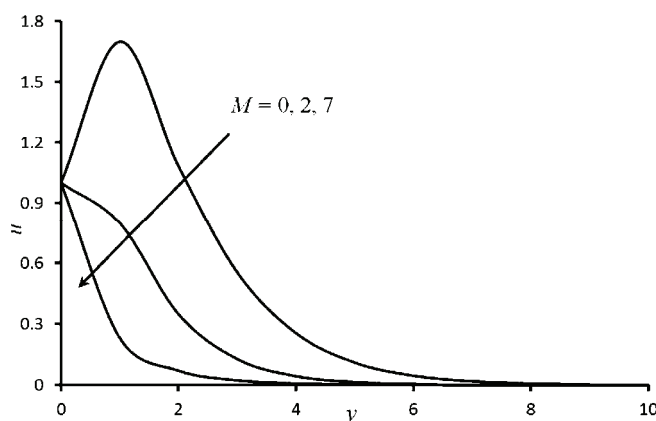


Fig. 3. Velocity for various values of M , when $Sc = 0.6$, $K_c = 1.0$, $a = 0.5$, $Pr = 2.0$, $S = 0.4$, $\beta = 1.13$, $K = 2.0$, $G_r = 5.0$, $G_c = 0.5$ and $Ec = 0.03$.

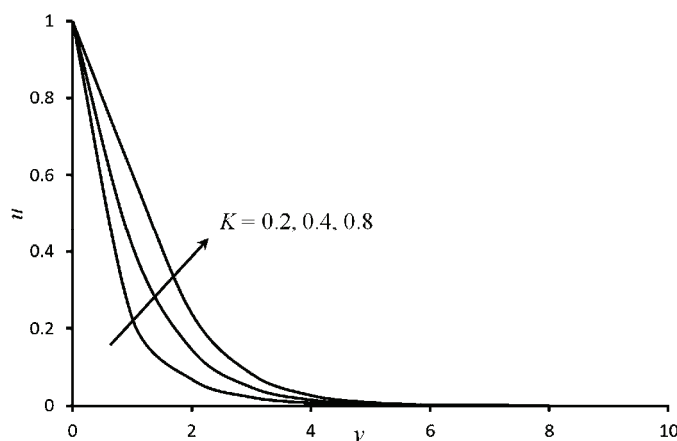


Fig. 4. Velocity for various values of K , when $Sc = 0.6$, $K_c = 1.0$, $a = 0.5$, $Pr = 2.0$, $S = 0.4$, $\beta = 1.13$, $M = 2.0$, $G_r = 5.0$, $G_c = 0.5$ and $Ec = 0.03$.

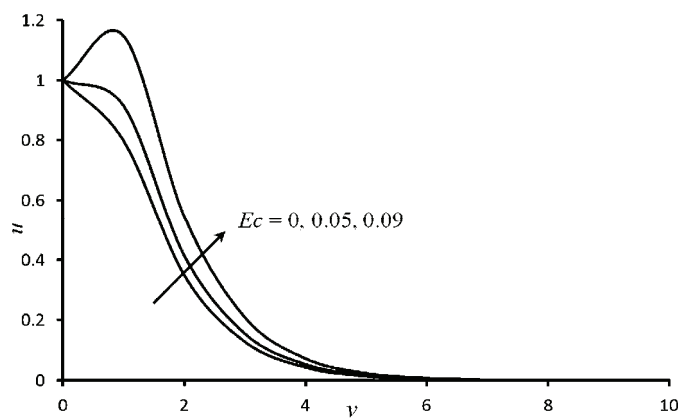


Fig. 5. Velocity for various values of Ec , when $Sc = 0.6$, $K_c = 1.0$, $a = 0.5$, $Pr = 2.0$, $S = 0.4$, $\beta = 1.13$, $M = 2.0$, $K = 2.0$, $G_r = 5.0$ and $G_c = 0.5$.

The effects of Ec on the angular velocity are plotted in Fig. 6. Ec increases with ω . With increasing rotational velocity, the shear stress due to the viscosity of the fluid creates a higher dissipation.

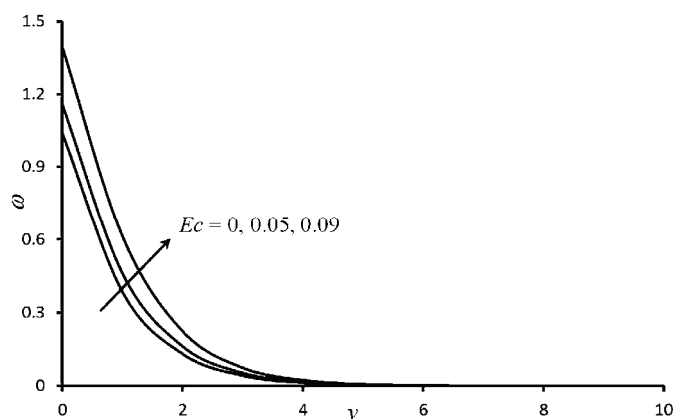


Fig. 6. Angular velocity for various values of Ec , when $Sc = 0.6$, $K_c = 1.0$, $a = 0.5$, $Pr = 2.0$, $S = 0.4$, $\beta = 1.13$, $M = 2.0$, $K = 2.0$, $G_r = 5.0$ and $G_c = 0.5$.

The fluid temperature for various values of Pr and S is shown in Figs. 7 and 8, respectively. θ increases with Pr and decreases as S increases. As the wall is at a prescribed heat flux, the temperature increases with increasing Pr due to the heat flux impinging on the surface. Whereas the presence of a heat source with a negative heat flux at the wall is the cause of a reduction in the fluid temperature.

Effects of magnetic field over the temperature are presented in Fig. 9. It was found that θ increases with M . This result is evidence for the fact that a magnetic

field increases the temperature of the fluid inside the boundary-layer because of excess heating.

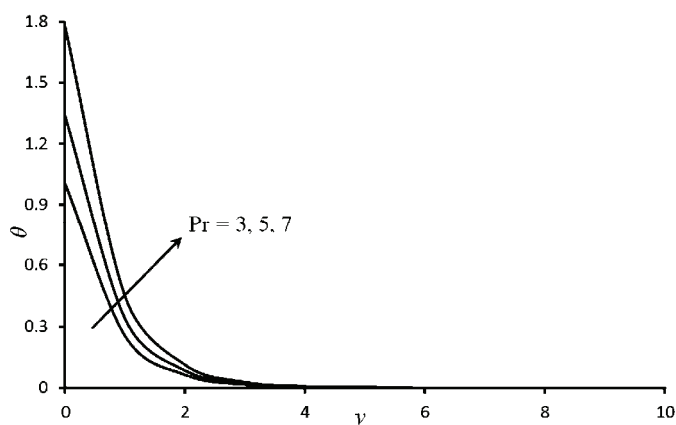


Fig. 7. Temperature for various values of Pr , when $Sc = 0.6$, $K_c = 1.0$, $a = 0.5$, $S = 0.4$, $\beta = 1$, $M = 2.0$, $K = 2.0$, $G_r = 5.0$, $G_c = 0.5$ and $Ec = 0.03$.

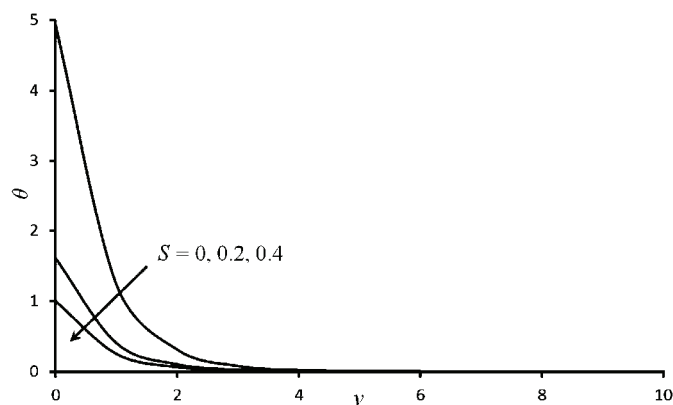


Fig. 8. Temperature for various values of S , when $Sc = 0.6$, $K_c = 1.0$, $a = 0.5$, $Pr = 3.0$, $\beta = 1$, $M = 2.0$, $K = 2.0$, $G_r = 5.0$, $G_c = 0.5$ and $Ec = 0.03$.

The effects of radiation and viscous dissipation are shown in Figs. 10 and 11. Figure 10 shows that θ decreases as β increases or N increases (as $\beta = 1 + 4/3N$). It was observed that an increase in the thermal radiation produces significant increases in the thermal condition of the fluid and the thickness of the thermal boundary layer. Moreover, Fig. 11 shows that Ec reduces the fluid temperature. An increasing Ec implies that dissipation of thermal energy is higher, which reduces the temperature.

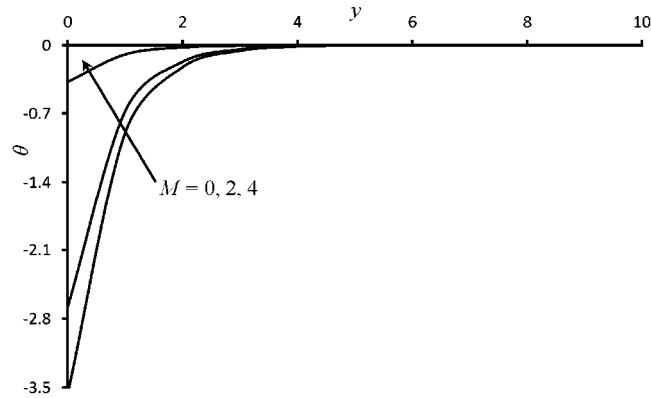


Fig. 9. Temperature for various values of M , when $Sc = 0.6$, $K_c = 1.0$, $a = 0.5$, $Pr = 2.0$, $S = 0.4$, $\beta = 1.13$, $K = 2.0$, $G_r = 5.0$, $G_c = 0.5$ and $Ec = 0.03$.

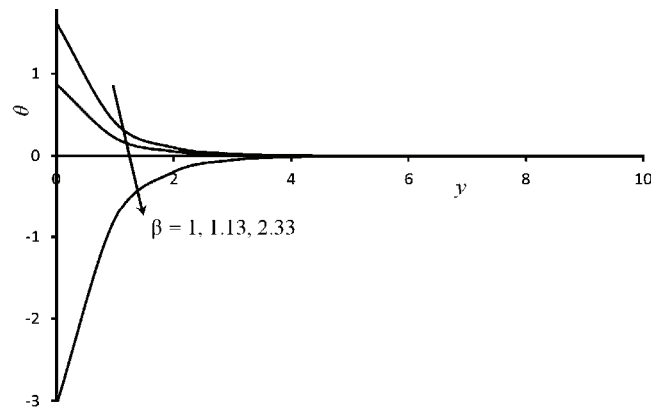


Fig. 10. Temperature for various values of β , when $Sc = 0.6$, $K_c = 1.0$, $a = 0.5$, $Pr = 3.0$, $S = 0.2$, $M = 2.0$, $K = 2.0$, $G_r = 5.0$, $G_c = 0.5$ and $Ec = 0.03$.

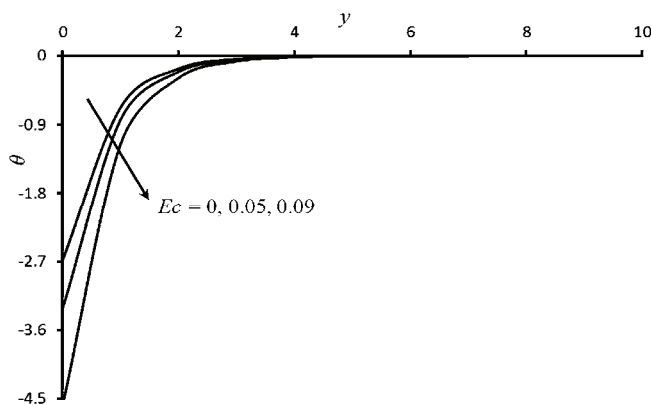


Fig. 11. Temperature for various values of Ec , when $Sc = 0.6$, $K_c = 1.0$, $a = 0.5$, $Pr = 2.0$, $S = 0.4$, $\beta = 1.13$, $M = 2.0$, $K = 2.0$, $G_r = 5.0$ and $G_c = 0.5$.

CONCLUSIONS

In this paper, the problem of MHD mixed convective flow of a micropolar fluid with the effect of ohmic heating, radiation and viscous dissipation over a chemically reacting porous plate was studied when the plate is at a constant heat flux. The effects of Schmidt number and chemical reaction is to reduce the concentration. The magnetic field reduces the velocity of the boundary layer but enhances the thermal boundary layer. The presence of porosity decreases the transverse velocity. Increasing fluid viscosity increases the fluid temperature. On the other hand, the presence of a heat source or radiation narrows the thermal boundary layer. The effect of viscous dissipation is to increase the transverse velocity and angular velocity, but to reduce the temperature.

NOMENCLATURE

y^*	horizontal coordinate (m)
u^*	axial velocity (m s^{-1})
v^*	transverse velocity (m s^{-1})
ω_a	angular velocity vector normal to the xy -plane (rad s^{-1})
p^*	pressure (Pa)
T^*	temperature of the fluid (K)
T_∞	far field temperature (K)
c	species concentration (mol m^{-3})
c_∞	far field concentration (mol m^{-3})
ν	kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)
ρ	density (kg m^{-3})
κ	vortex viscosity (Pa s)
μ	dynamic viscosity (Pa s)
g	acceleration due to gravity (m s^{-2})
β_T	coefficient of thermal expansion (K^{-1})
β_c	coefficient of concentration expansion ($\text{m}^3 \text{mol}^{-1}$)
K^*	permeability of porous medium (H m^{-1})
σ	electrical conductivity (S m^{-1})
B_0	magnetic field coefficient (T)
J	microinertia density (m^2)
γ	spin gradient viscosity (kg m s^{-1})
c_p	specific heat ($\text{J kg}^{-1} \text{K}^{-1}$)
k	thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)
q_r^*	radiative heat flux in the y -direction (W m^{-2})
D	mass diffusion coefficient ($\text{m}^2 \text{s}^{-1}$)
K_l	rate of chemical reaction (s^{-1})
q	rate of heat transfer (W m^{-2})
m_w	wall mass flux ($\text{mol m}^{-2} \text{s}^{-1}$)
σ'	Stefan–Boltzmann constant ($\text{W m}^{-2} \text{K}^{-4}$)
κ^*	mean absorption coefficient (m^{-1})
V_0	suction velocity (m s^{-1})
Q	heat generation coefficient ($\text{W m}^{-3} \text{K}^{-1}$)

a	material parameter
y	dimensionless horizontal coordinate
u	dimensionless axial velocity
M	magnetic field parameter
Pr	Prandtl number
θ	dimensionless temperature
C	dimensionless species concentration
Ec	Eckert number
G_r	thermal Grashof number
G_c	solutal Grashof number
S	heat generation parameter
ω	dimensionless angular velocity
Sc	Schmidt number
N	radiation parameter
K_c	chemical reaction parameter
K	dimensionless permeability parameter.

Acknowledgment. The author is very much thankful to Prof. (Dr.) S. S. Tak, Jai Narain Vyas University, Jodhpur (India) for offering his valuable suggestions and assistance, and to the learned referee for significant and valuable comments.

ИЗВОД

МЕШОВИТИ КОНВЕКТИВНИ-МАГНЕТОХИДРОДИНАМИЧКИ (МНД) ТОК
МИКРОПОЛАРНОГ ФЛУИДА СА ОМСКИМ ГРЕЈАЊЕМ, ЗРАЧЕЊЕМ И ВИСКОЗНОМ
ДИСИПАЦИЈОМ, ПРЕКО ХЕМИЈСКИ РЕАКТИВНЕ ПОРОЗНЕ ПЛОЧЕ ИЗЛОЖЕНЕ
КОНСТАНТНОМ ТОПЛОТНОМ ФЛУКСУ И КОНЦЕНТРАЦИОНОМ ГРАДИЈЕНТУ

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У овом раду је приказана анализа мешовите конвекције-магнетохидродинамичког (МНД) микрополарног тока са хемијском реакцијом, преноса топлоте и масе у порозној средини са ефектима омског грејања, зрачења и вискозне дисипације, преко бесконачне вертикалне плоче која је изложена константном топлотном флуксу и концентрационом градијенту. Нелинеарне купловане парцијалне диференцијалне једначине су решаване нумерички коришћењем имплицитне схеме коначних разлика, познате као *Keller box* метод. Резултати за концентрације, трансверзалну и угаону брзину и температуре су добијени и графички илустровани да би се уочили ефекти појединих параметара. Нумеричка дискусија са физичким интерпретацијама је такође приказана.

(Примљено 12. јуна, ревидирано 11. октобра 2013)

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