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An analytical solution to the problem of radiative heat and mass transfer over an inclined plate at a prescribed heat flux with chemical reaction

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Abstract: A steady laminar flow of viscous electrically conducting incompressible fluid over a semi-infinite inclined porous plate, which was at a prescribed heat flux, with radiation, heat generation and chemical reaction is presented in this manuscript. The analytical solutions for velocity, concentration and temperature were found in terms of an exponential function. The effects of various parameters, such chemical reaction, thermal Grashof number, radiation parameter, angle of inclination, *etc*., on the velocity and temperature are presented graphically.

Keywords: heat transfer; mass transfer; heat flux; heat generation; concentration; inclined wall; chemical reaction.

INTRODUCTION

Convection flow driven by temperature and concentration differences has been the objective of extensive research because such processes exist in nature and has engineering applications. The process occurring in nature includes the photosynthetic mechanism, calm-day evaporation and vaporization of mist and fog, while the engineering applications include chemical reaction in a reactor chamber consisting of rectangular ducts, chemical vapor deposition on surfaces and cooling of electronic equipment. The study of natural convection flow for an incompressible viscous fluid past a heated surface has important applications, such as the cooling of nuclear reactors, the boundary layer control in aerodynamics, crystal growth and food processing and cooling towers.

Moreover, when the temperature of the surrounding fluid is high, radiation effects play an important role that cannot be ignored.[1,2] The effects of radiation on temperature have become more important industrially. Many processes in engineering areas occur at high temperatures and in such cases radiative heat

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transfer become important for the design of pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. In such cases, the effects of radiation and free convection have to be taken into consideration. For an impulsively started infinite vertical isothermal plate, Ganesan *et al.*[3] studied the effects of radiation and free convection by using Rosseland approximation.[4] The problem of radiative heat transfer with hydromagnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux was solved analytically by Kumar.[5]

The problems of free convection and mass transfer of an electrically conducting fluid past an inclined surface with the effect of a magnetic field has vast applications in geophysics, astrophysics and many other engineering areas. Chen[6] studied the analysis of natural convection flow over a permeable inclined surface with variable wall temperatures and concentration. Hossain *et al.*[7] studied free convection flow from an isothermal plate at a small angle to the horizontal. Anghel *et al.*[8] presented a numerical solution of free convection flow past an inclined surface. Bhuvaneswari *et al.*[9] studied an exact analysis of radiation convective heat and mass transfer flow over an inclined plate in a porous medium. Sivasankaran *et al.*[10] presented a Lie group analysis of natural convection heat and mass transfer in an inclined surface. In many engineering and physical problems in which a fluid undergoes exothermic or endothermic reaction, it is highly important to study the effect of heat generation and absorption. Therefore, the study of heat generation or absorption of a moving fluid is important in chemical reactions and those concerned with dissociating fluids. Chamkha and Khalid[11] introduced similarity solutions for hydromagnetic simultaneous heat and mass transfer with heat generation and absorption in natural convection from an inclined plate. Vajravelu and Hadjinicolaou[12] studied the heat transfer boundary layer of a viscous fluid over a stretching sheet with internal heat generation. Kumar[13] investigated heat transfer over a stretching porous sheet subjected to a power law heat flux in the presence of a heat source.

Diffusion rates can be altered tremendously by chemical reactions. The effect of a chemical reaction depends on whether the reaction is homogeneous or heterogeneous. This depends on whether they occur in an interface or as a single-phase volume reaction. In a well-mixed system, the reaction is heterogeneous if the reactants are in multiple phases and homogeneous if the reactants are in the same phase. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. Kandasamy *et al.*[14] studied thermophoresis and variable viscosity effects on the magnetohydrodynamics of mixed convective heat and mass transfer past a porous wedge in the presence of a chemical reaction. Kandasamy and Devi[15] studied the effects of chemical reaction, heat and mass transfer on non-linear laminar boundary-layer flow over a wedge with suction or injection.

In the present work, radiative heat and mass transfer over an inclined plate in the presence of a chemical reaction was studied when the wall is at prescribed heat flux. The effects of the chemical reaction, radiation parameters, the thermal Grashof number and the angle of inclination on the velocity and temperature fields were studied.

MATHEMATICAL ANALYSIS

Consider a steady laminar flow of an incompressible viscous electrically conducting fluid past a semi-infinite inclined porous wall with an acute angle ϕ from the vertical in the presence of a chemical reaction and radiation. The wall is at the prescribed heat flux. The flow is assumed to be in the x-direction, which is taken along the semi-infinite inclined porous plate with the y-axis normal to it. A magnetic field of uniform strength B_0 is introduced normal to the flow direction. In the analysis, it is assumed that the magnetic Reynolds number is much lower than unity so that the induced magnetic field can be neglected in comparison to the applied magnetic field. It is also assumed that all fluid properties are constant. Then, under the usual Boussinesq and boundary layer approximations, the governing equations of the mass, momentum, energy and concentration for steady flow can be written as:

$$\frac{\partial v}{\partial y}=0 \tag{1}$$

$$v\frac{\partial u}{\partial y}=\vartheta\frac{\partial^2 u}{\partial y^2}+g\,\beta_T\,(T-T_\infty)\cos\varphi+g\,\beta_C\,(c-c_\infty)\cos\varphi-\frac{\vartheta}{K'}u-\frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{Q}{\rho c_p}\left(T-T_\infty\right) \tag{3}$$

$$v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2}-K_l\left(c-c_\infty\right) \tag{4}$$

with the boundary conditions:

$$u=0\,,\ v=-v_{\mathrm{w}}\,,\ \frac{\partial T}{\partial y}=-\frac{q}{k}\,,\ c=c_{\mathrm{w}}\,,\ \text{at}\ \ y=0$$

$$u\to 0\,,\ T\to T_\infty\,,\ c\to c_\infty\,,\ \text{as}\ \ y\to\infty \tag{5}$$

$$v_{\mathrm{w}}>0$$

The equation of continuity (1) with the boundary condition (5) changes to:

$$v=-v_{\mathrm{w}} \tag{6}$$

here, $v_{\mathrm{w}}>0$.

Assuming the Rosseland approximation[4] for the radiative heat flux leads to:

$$q_r=-\frac{4\sigma'}{3\kappa^*}\frac{\partial T^4}{\partial y}$$

If the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature, then the Taylor series for T^4 about T_∞, after ignoring higher order terms, is given by:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4$$

Assuming the non-dimensional variables are as follows:

$$Y = \frac{yv_w}{\vartheta},\ U = \frac{u}{u_w},\ \theta = \frac{k v_w (T - T_\infty)}{q\vartheta},\ C = \frac{c - c_\infty}{c_w - c_\infty}, Gr_T = \frac{g\beta_T q\vartheta^2}{k u_w v_w^3},$$
$$Gr_c = \frac{g\beta_c (c_w - c_\infty)\,\vartheta}{u_w v_w^2},\ K = \frac{K' v_w^2}{\vartheta^2},\ M^2 = \frac{B_0^2 \vartheta\sigma}{v_w^2 \rho},\ \mathrm{Pr} = \frac{\rho\vartheta c_p}{k} = \frac{\vartheta}{\alpha},$$
$$N = \frac{\kappa_* k}{4\sigma' T_\infty^3},\ S = \frac{Q\vartheta}{\rho c_p v_w^2},\ Sc = \frac{\vartheta}{D}\ \text{and}\ K_c = \frac{\vartheta K_l}{v_w^2}$$

and using these non-dimensional parameters, Eqs. (2)–(4) are reduced to:

$$\frac{\mathrm{d}^2 U}{\mathrm{d}Y^2} + \frac{\mathrm{d}U}{\mathrm{d}Y} + Gr_T\,\theta\cos\varphi + Gr_c\,C\cos\varphi - \frac{1}{K}U - M^2 U = 0 \tag{7}$$

$$\left(1 + \frac{4}{3N}\right)\frac{\mathrm{d}^2\theta}{\mathrm{d}Y^2} + \mathrm{Pr}\frac{\mathrm{d}\theta}{\mathrm{d}Y} + \mathrm{Pr}\,S\,\theta = 0 \tag{8}$$

$$\frac{\mathrm{d}^2 C}{\mathrm{d}Y^2} + Sc\frac{\mathrm{d}C}{\mathrm{d}Y} - K_c\,Sc\,C = 0 \tag{9}$$

With the corresponding boundary conditions:

$$\begin{aligned} &U = 0,\ \frac{\partial\theta}{\partial Y} = -1,\ C = 1,\ \text{at}\ Y = 0 \\ &U \to 0,\ \theta \to 0,\ C \to 0,\ \text{as}\ Y \to \infty \end{aligned} \tag{10}$$

the solutions of Eqs. (7)–(9) with boundary condition (10) are as follows:

$$C = \mathrm{e}^{-aY} \tag{11}$$

$$\theta = d\,\mathrm{e}^{-bY} \tag{12}$$

$$U = p_1\mathrm{e}^{-fY} - p_2\mathrm{e}^{-bY} - p_3\mathrm{e}^{-aY} \tag{13}$$

where:

$$a = \frac{Sc + \sqrt{Sc^2 + 4K_c Sc}}{2},\ L^2 = M^2 + \frac{1}{K},\ \omega = 1 + 4/(3N),\ b = \frac{Pr + \sqrt{Pr^2 - 4Pr\,S\omega}}{2\omega},$$
$$d = \frac{1}{b},\ f = \frac{1 + \sqrt{1 + 4L^2}}{2},\ p_2 = \frac{Gr_T\,d\cos\varphi}{b^2 - b - L^2},\ p_3 = \frac{Gr_c\cos\varphi}{a^2 - a - L^2}\ \text{and}\ p_1 = p_2 + p_3$$

The wall shear stress is given by:

$$\tau_f = \mu\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)_{y=0} \tag{14}$$

and the skin friction coefficient is defined as:

$$C_f = \frac{\tau_f}{\rho u_w v_w} = \left(\frac{\mathrm{d}U}{\mathrm{d}Y}\right)_{Y=0} = -fp_1 + bp_2 + ap_3 \tag{15}$$

The recovery factor can be written as:

$$R_f = \theta(0) = d \tag{16}$$

the wall mass transfer rate is:

$$J_w = -D\left(\frac{\partial C}{\partial Y}\right)_{Y=0} \tag{17}$$

and the Sherwood number is defined as:

$$Sh = \frac{J_w \vartheta}{(C_w - C_\infty) D v_w} = -\frac{\mathrm{d}C}{\mathrm{d}Y}(0) = a \tag{18}$$

RESULTS AND DISCUSSION

A study of the velocity, temperature, concentration, skin friction, recovery factor and Sherwood number of the steady laminar flow of an incompressible viscous electrically conducting fluid past a semi-infinite inclined wall was performed in the present research.

The velocity as a function of K_c. is plotted Fig. 1 and the obtained results were compared with those of Kandasamy and Devi[15] and good agreement was found. It can be seen from Fig. 1 that the velocity the fluid decreased with increasing chemical reaction, which was considered to be a homogeneous first-order chemical reaction. The diffusing species can either be destroyed or generated in the homogeneous reaction. The chemical reaction parameter can be adjusted to meet these circumstances if one takes $K_c > 0$ for a destructive reac-

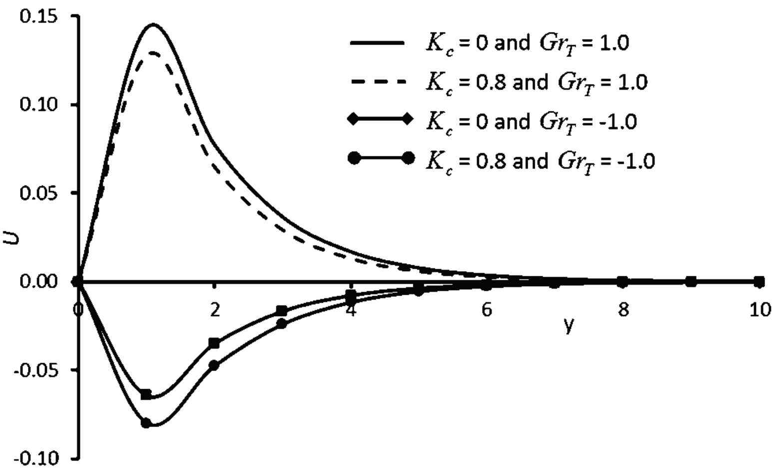

Fig. 1. Dimensionless velocity against non-dimensional y for different values of K_c and Gr_T when $Sc = 0.78$, $Pr = 1.0$, $S = 0.1$, $\omega = 1.13$, $K = 1.0$, $M = 1.73$, $\phi = 30°$ and $Gr_c = 0.5$.

tion, $K_c < 0$ for a generative reaction and $K_c = 0$ for no reaction. A destructive chemical reaction was assumed herein.

The effect of the angle of inclination (ϕ) on the velocity is shown in Fig. 2. The velocity decreases as ϕ increases. The fluid has a higher velocity when the surface is vertical than when it is inclined because the buoyancy effect decreases due to gravity components ($g\cos\phi$) as the plate is inclined. In the case of $Gr_T < 0$, it can be seen that ϕ increases the velocity. This is because a negative Gr_T makes the rate of heat transfer also negative and at wall, the heat flux becomes positive, which accelerates the convection effect at the wall; the stream function for lower values of the thermal Grashof number become thinner (diluted) due to stronger convection, hence the velocity decreases.

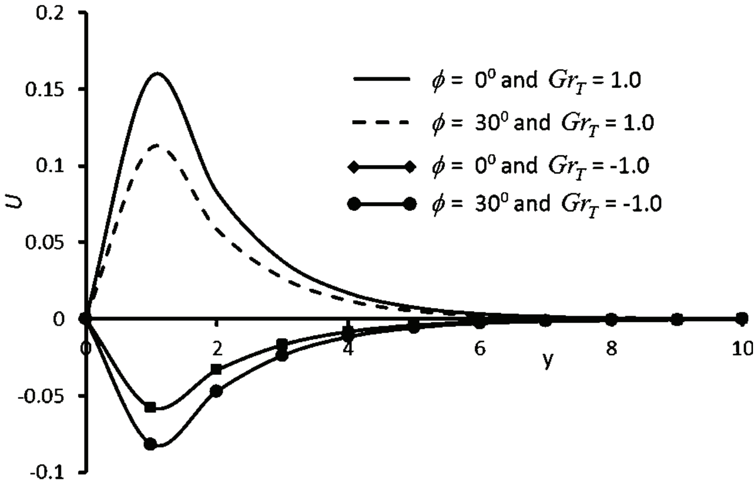

Fig. 2. Dimensionless velocity against non-dimensional y for different values of ϕ and Gr_T when $Sc = 0.78$, $K_c = 0.2$, $Pr = 1.0$, $S = 0.1$, $\omega = 1.13$, $K = 1.0$, $M = 1.73$ and $Gr_c = 0.5$.

The effects of ω on temperature is presented in Fig. 3, from which it can be seen that θ increases as ω increases or N decreases (because $\omega = 1 + 4/3N$). Numerically, increasing the radiation parameter reduces the radiation effect; physically, increasing the radiation parameter leads to a decrease in the thickness of the thermal boundary layer.

CONCLUSIONS

In the present study, an analytical solution of steady hydromagnetic boundary-layer flow over a semi infinite inclined plate, which is at prescribed heat flux, in the presence of chemical reaction, buoyancy, heat generation and thermal radiation. The flow equations were solved analytically and the obtained results were compared with earlier published work and found to be in good agreement. The velocity of the fluid decreases as the chemical reaction or angle of inclination increases. Radiation (increasing with 1/N) extends the thermal boundary layer and decreases the heat transfer from the surface to the fluid.

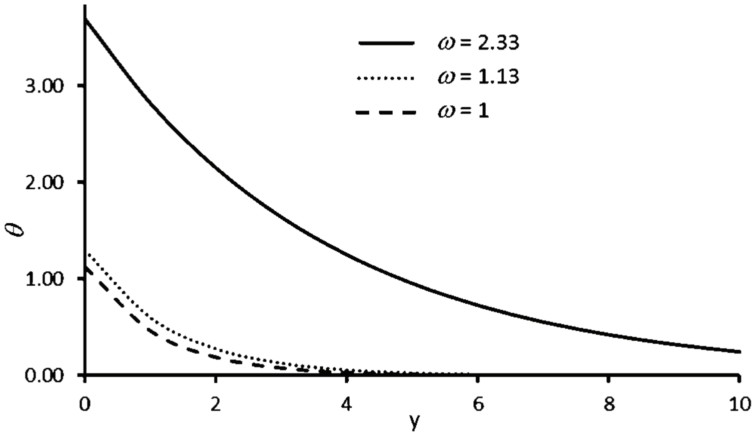

Fig. 4. Dimensionless temperature against non-dimensional *y* for different values of *ω* when *Pr* = 1.0 and *S* = 0.1.

NOMENCLATURE

y	horizontal coordinate (m)
u	axial velocity (m s^{-1})
v	transverse velocity (m s^{-1})
T	temperature of the fluid (K)
T_∞	far field temperature (K)
c	species concentration (mol m^{-3})
c_∞	far field concentration (mol m^{-3})
c_w	concentration on the surface (mol m^{-3})
g	acceleration due to gravity (m s^{-2})
β_T	coefficient of thermal expansion (K^{-1})
β_C	coefficient of concentration expansion (m^3 mol^{-1})
ϕ	angle of inclination (°)
ϑ	kinematic viscosity (m^2 s^{-1})
K'	permeability of porous medium (m^2)
σ	electrical conductivity (S m^{-1})
B_0	magnetic field coefficient (T)
α	thermal diffusivity (m^2 s^{-1})
Q	heat generation coefficient (W m^{-3} K^{-1})
ρ	density (kg m^{-3})
c_p	specific heat (J Kg^{-1} K^{-1})
q	radiative heat flux in the *y*-direction (W m^{-2})
D	mass diffusion coefficient (m^2 s^{-1})
K_l	rate of chemical reaction (s^{-1})
q	rate of heat transfer (W m^{-2})

k thermal conductivity (W m^{-1} K^{-1})
u_w surface velocity (m s^{-1})
v_w suction velocity (m s^{-1})
σ' Stefan–Boltzmann constant (W m^{-2} K^{-4})
K^* mean absorption coefficient (m^{-1})
Gr_T thermal Grashof number
Gr_c solutal Grashof number
K dimensionless permeability parameter
M^2 magnetic field parameter
Pr Prandtl number
N radiation parameter
S heat generation parameter
Sc Schmidt number
K_c chemical reaction parameter
Y dimensionless horizontal coordinate
U dimensionless axial velocity
Θ dimensionless temperature
C dimensionless species concentration
Constants: a, b, d, ω, f, p_1, p_2, p_3 and L^2.

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И З В О Д

АНАЛИТИЧКО РЕШЕЊЕ ПРОБЛЕМА ПРЕНОСА ТОПЛОТЕ ЗРАЧЕЊЕМ И ПРЕНОСА МАСЕ СА ХЕМИЈСКОМ РЕАКЦИЈОМ НА КОСОЈ ПЛОЧИ ПРИ ЗАДАТОМ ФЛУКСУ ТОПЛОТЕ

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Математички модел стационарног ламинарног тока вискозног, електропроводног, нестишљивог флуида, преко полу-бесконачне косе порозне плоче, са задатим флуксом топлоте, са зрачењем, генерисањем топлоте и хемијском реакцијом, је приказан у раду. Аналитичка решења за брзину, концентрацију и температуру су добијена у облику експоненцијалних функција. Утицаји различитих параметара, као што су хемијска реакција, топлотни Грасхофов број, параметар зрачења, угао нагиба плоче, итд., на брзину и температуру су приказани графички.

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