



J. Serb. Chem. Soc. 76 (11) 1567–1581 (2011)
JSCS–4230

An analytical study of the effects of vaporization of two-dimensional laminar droplets on a triple flame

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(Received 13 August 2010, revised 10 January 2011)

Abstract: The structure of triple flame propagation in combustion systems, containing uniformly distributed volatile fuel droplet was analyzed. The analysis was established for a one-step irreversible reaction with an asymptotic limit, where the value of the Zeldovich number is large. Here, using unit Lewis number, the analytical results for the triple flame temperature were obtained considering two sections. In the first section, a non-vaporizing fuel stream was studied and in the second section, a volatile droplet fuel stream was taken into account. It is presumed that the fuel droplets vaporize to yield a gaseous fuel of known chemical structure, which is subsequently oxidized in the gaseous phase. Here two different cases are studied. In the first case, only the velocity parallel to the reactant flow was considered; while for the latter one, the vertical velocity was considered in addition. The energy equations were solved and the temperature field equations are presented. The results are first presented for a non-vaporizing fuel and compared to the experiment results. In addition, some other results of the temperature field for a vaporizing fuel stream are demonstrated within the comparison between the above-mentioned cases which revealed the effect of the considering the vertical velocity component on the flame temperature field.

Keywords: triple-flame; volatile droplet; flame temperature.

INTRODUCTION

The importance of triple flames is now well established in applications involving combustion phenomena, such as flame spread over solid or liquid fuel surfaces, flame propagation in mixing layers, dynamic extinction of diffusion flames and flame stabilization in reactive streams.

A flame propagating through a fuel/air mixing layer, mainly when the fuel concentration is ranged from lean to rich, may display a triple flame structure, which is composed of a lean and a rich premixed flame wing with a trailing dif-

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doi: 10.2298/JSC100813133B

fusion flame, all extending from a single point. Mixing layers are frequently encountered in combustion problems, including two-dimensional (2-D) mixing layers, heterogeneous propellant combustion, opposed flame spread, jets and boundary layers. The schematic of a triple flame is shown in Fig. 1.

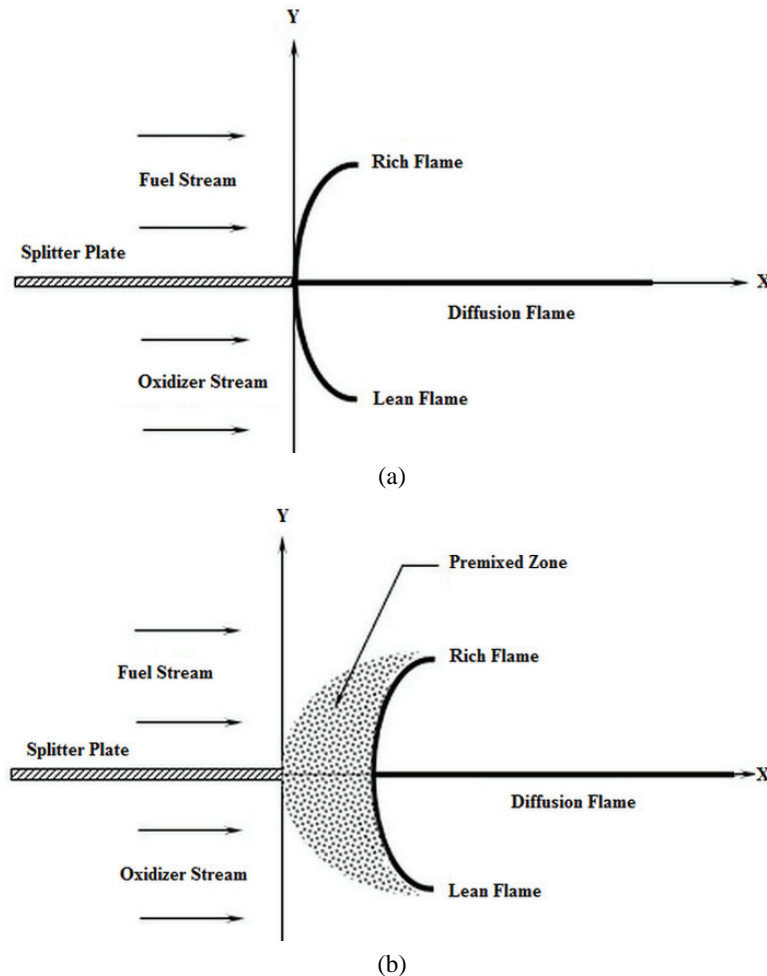


Fig. 1. Triple flame schematic; a) the assumed configuration for the present work; b) the configuration which existed in reality.

The stabilization, propagation and instability of triple flames were studied by Chung.¹ The prospect of establishing a planar turbulent triple flame was considered in the work of Bray and Champion.² The local flame speed of a two-dimensional methane-air triple flame in a rectangular burner was investigated by Hirota *et al.*³ The effect of reversibility of the chemical reaction on triple flame

propagation was studied by Ali and Daou.⁴ The effects of hydrogen enrichment on the propagation of laminar methane–air triple flames in axisymmetric co-flowing jets were numerically investigated by Briones *et al.*⁵ A numerical study of a laminar methane–air triple flame in two-dimensional mixing layers was performed by Guo *et al.*⁶ The effect of flow strain on triple flame propagation was investigated by Hong and Chen.⁷ Flame spread in laminar mixing layers was investigated both experimentally and numerically by Kioni *et al.*⁸ The dynamics of a laminar triple flame was numerically investigated for different degrees of mixture by Dobrego *et al.*⁹ The interaction of non-unity Lewis numbers (due to preferential diffusion and/or unequal rates of heat and mass transfer, was investigated by Shamim.¹⁰

The aim of this work is to extend current knowledge of triple flames by taking into consideration the influence of a vaporization term for the fuel droplets. This aspect of the problem seems to have received no attention. The purpose of this paper was to investigate how triple flames, and their propagation regimes, are affected by the vaporization term of perfectly volatile fuel droplets in this configuration. In addition, the influence of flow velocity components on the triple flame temperature field was studied.

The paper is structured as follows: the problem is first formulated for a non-vaporizing fuel stream, as a one-step combustion process considering constant density and constant conductivity properties as well as an equal ratio of mass diffusivity to temperature diffusivity (unit Lewis number). The basic equations are solved in terms of dimensionless variables with specified dimensionless boundary conditions. The results are then compared to the experimental work.¹¹ Then, using the same approach mentioned above, a vaporizing fuel stream will be studied for two different cases related to the flow velocity components. The results obtained in both cases are described and compared.

FORMULATIONS FOR A NON-VAPORIZING FUEL STREAM

The problem addressed herein is the steady propagation of a non-premixed flame consisting of fuel and oxidizer streams. All external forces, including gravitational effects, are assumed to be negligible.

Other approximations introduced are that diffusion caused by a pressure gradient and heat transport by radiation are negligible. The combustion process is modeled by a single, irreversible, one-step reaction of the form $v_F F + v_X X \rightarrow v_P P$, where v_F molecules of a fuel (molecular mass m_F) react with v_X molecules of an oxidizer (molecular mass m_X) to form v_P product molecules (molecular mass m_P). At the first step in the analysis, the problem will be solved within the approximations above. The reactants and products are assumed to be in the gaseous phase. For simplicity and in order to be able to compare the results with the experiments, in this paper, only the case of unity Lewis number, *i.e.*, the rate of temperature diffusivity and mass diffusivity of the two species are the same, will be considered.

Basic equations

The time evolution of the mass fractions of the two reactants, Y_F (the fuel mass fraction) and Y_X (the oxidizer mass fraction) is described by:¹²

$$\rho \frac{DY_i}{Dt} = D_i \nabla^2 Y_i - v_i m_i \omega \quad (1)$$

where $I = F$ or X , $D / Dt = \partial / \partial t + u \nabla$ is the material derivative, ρ and u are the fluid density and velocity, respectively, ω is the rate of reaction in unit volume per unit time, and D_i is the mass diffusivity of the species i . The time evolution of the temperature, T , is described by:

$$\rho c_p \frac{DT}{Dt} = D_T \nabla^2 T + Q \omega \quad (2)$$

where c_p is the specific heat at constant pressure referred to unit mass of gas, Q is the amount of heat released per reaction and D_T is the thermal conductivity. The Lewis numbers will be assumed unity, thus:

$$\frac{D_F}{\rho_\infty} = \frac{D_X}{\rho_\infty} = \frac{D_T}{\rho_\infty c_p} \equiv k \quad (3)$$

where ρ_∞ is the density of un-burnt gas far upstream, which is assumed to be constant. Therefore Eqs. (1) and (2) become:

$$\rho \frac{DY_i}{Dt} = k \rho_\infty \nabla^2 Y_i - v_i m_i \omega \quad (4)$$

and

$$\rho c_p \frac{DT}{Dt} = k \rho_\infty c_p \nabla^2 T + Q \omega \quad (5)$$

For the reaction rate, the well known Arrhenius law will be applied by¹²

$$\omega = A \rho^{v_F + v_X} Y_F^{v_F} Y_X^{v_X} \exp(-T_a / T) \quad (6)$$

where T_a is the activation temperature, which is constant for a given reaction, and the exponential factor A is assumed to be approximately constant and has no temperature dependence.

The problem is considered as a uniform flow along the x -axis between $x=(-l, l)$, by $U_\infty x$, where U_∞ is the propagation speed of the triple flame far upstream. It is assumed that the fuel mass fraction varies from $Y_F = 1$ in the fuel stream ($y \rightarrow +L$) to $Y_F = 0$ in the oxidizer stream ($y \rightarrow -L$). Similarly, the oxidizer mass fraction varies from $Y_X = 1$ in oxidizer stream ($y \rightarrow -L$) to $Y_X = 0$ in fuel stream ($y \rightarrow +L$), *i.e.*:

$$\begin{aligned} y \rightarrow +L & \quad Y_F = 1 \quad Y_X = 0 \\ y \rightarrow -L & \quad Y_F = 0 \quad Y_X = 1 \end{aligned} \quad (7)$$

such that for upstream:

$$Y_F(-l, y) + Y_X(-l, y) = 1 \quad (8)$$

The fuel and oxidizer are assumed to be at the same constant temperature far upstream, $T = T_\infty$. Therefore, the boundary conditions in a limited domain are:

$$\begin{aligned} T(-l, y) &= T_\infty \\ T(0, y) &= T_s \\ T_y(x, 0) &= 0 \\ T(x, -L) &= T_\infty \end{aligned} \quad (9)$$

The fuel mixture fraction, Z , is then defined as:¹³

$$Z = \frac{rY_F - Y_X + 1}{1+r} \quad (10)$$

where $r = \nu_X m_X / \nu_F m_F$. Clearly, $Z = 1$ in the fuel stream, $Z = 0$ in the oxidizer stream and $Z = Z_s = 1/(1+r)$ under stoichiometric condition.

It follows from Eqs. (5) and (10) that Z evolves as a passive scalar:

$$\rho \frac{DZ}{Dt} = k \rho_\infty \nabla^2 Z \quad (11)$$

By eliminating the source terms as well as the vaporization terms from Eqs. (4) and (6), it can be shown that the specific enthalpy, which is defined below by Eq. (12), also evolves as a passive scalar (Eq. (13)):

$$H_i = T + \frac{QY_i}{\nu_i m_i c_p} \quad (12)$$

$$\rho \frac{DH_i}{Dt} = k \rho_\infty \nabla^2 H_i \quad (13)$$

From Eq. (10) and (11) and the condition that $T = T_\infty$ far upstream, H_i can be expressed as a linear function of Z as $H_i = A_i + B_i Z$, where A_i and B_i are constants determined from the conditions $Y_F = 1$ in the fuel stream and $Y_F = 0$ in the oxidizer stream. Thus, using the definition of H_i , the reactants mass fractions, Y_F and Y_X , may be expressed in terms of Z and T as follows:

$$Y_F = (T_\infty - T) \frac{m_F \nu_F c_p}{Q} + Z \quad (14)$$

$$Y_X = (T_\infty - T) \frac{m_X \nu_X c_p}{Q} - Z + 1 \quad (15)$$

Dimensionless system of the governing equations

In order to express the basic equations in terms of dimensionless variables, suitable dimensionless parameters have to be identified so that the magnitude of the parameters will be of the order unity. Thus, the dimensionless density $\bar{\omega} = \rho/\rho_\infty$ and the dimensionless velocity components $(U, V) = (u/U_\infty, v/U_\infty)$, as well as dimensionless coordinates $(X, Y) = (x U_\infty/k, y U_\infty/k)$. The maximum temperature is reached just behind the flame front along the stoichiometric line, where the combustion is complete and neither fuel nor oxidizer are left in the product stream. This ‘‘adiabatic flame temperature’’, T_s , is obtained by substituting $Z = Z_s = 1/(1+r)$ and $Y_F = 0$ in Eq.(14):

$$T_s = T_\infty + \frac{Q}{m_F \nu_F c_p} \frac{1}{1+r} = T_\infty + \frac{Q}{(m_F \nu_F + m_X \nu_X) c_p} \quad (16)$$

The dimensionless parameter α characterizes the temperature rise, or the amount of heat released in the flame:

$$\alpha \equiv \frac{T_s - T_\infty}{T_s} = \left[1 + \frac{(m_F V_F + m_X V_X) c_p T_\infty}{Q} \right]^{-1} \quad (17)$$

Further, the dimensionless temperature, Θ , and the Zeldovich Number, β , are introduced, which are defined as:

$$\Theta = \frac{T - T_\infty}{T_s - T_\infty} \quad (18)$$

$$\beta \equiv \alpha \frac{T_a}{T_s} \quad (19)$$

In terms of dimensionless variables, Eq. (5) may be written as:

$$\bar{\omega} U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} + \lambda \bar{\omega}^{V_F + V_X} \Sigma(Z / Z_s, \Theta) \exp \left[-\frac{\beta(1 - \Theta)}{1 - \alpha(1 - \Theta)} \right] \quad (20)$$

where Σ is defined by the following expression:¹³

$$\Sigma(x, y) \equiv (x - y)^{V_F} [(1 - x) + r(1 - y)]^{V_X} \quad (21)$$

and λ is defined by:

$$\lambda = \frac{kQA}{c_p T_s \alpha} \frac{\rho_\infty^{V_F + V_X - 1}}{(1 + r)^{V_F + V_X}} \exp(-\beta / \alpha) \frac{1}{U_\infty^2} \quad (22)$$

The dimensionless boundary conditions therefore become:

$$\begin{aligned} \Theta(-l, y) &= 0 \\ \Theta(0, y) &= 1 \\ \Theta_y(x, 0) &= 0 \\ \Theta(x, -L) &= 0 \end{aligned} \quad (23)$$

Activation energy asymptotic

In order to obtain an analytical solution, some approximations are introduced, whereby $\bar{\omega}$ may be taken unity. In addition, it is assumed $U = 1$ and $V = 0$.

Equation (20) thus becomes:

$$\frac{\partial \Theta}{\partial X} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} + \lambda \Sigma(Z / Z_s, \Theta) \exp[-\beta(1 - \Theta)] \quad (24)$$

The Zeldovich Number, β , defined by Eq. (19) is considered large for many practical processes. Thus, the basic dimensionless equations describing flames are studied in the asymptotic limit of $\beta \rightarrow \infty$. This limit is known as the activation energy asymptotic (AEA). It is clear from Eq. (24) that the source term vanishes and becomes:

$$\frac{\partial \Theta}{\partial X} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \quad (25)$$

Equation (25) is a linear homogeneous partial differential equation which is converted to Helmholtz differential equation by changing the variable Θ to ψ by the following substitution:¹⁴

$$\Theta(X, Y) = \exp\left(\frac{1}{2}X\right)\psi(X, Y) \quad (26)$$

Hence, Eq. (25) becomes:

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} - \frac{1}{4}\psi = 0 \quad (27)$$

Equation (27) is a two-dimensional homogeneous Helmholtz equation in the rectangular Cartesian system of coordinates, which may be solved by the separation of variables method. The boundary conditions also become:

$$\begin{aligned} \psi(-l, y) &= 0 \\ \psi(0, y) &= 1 \\ \psi_y(x, 0) &= 0 \\ \psi(x, -L) &= 0 \end{aligned} \quad (28)$$

Therefore,

$$\psi = \sum_{\kappa_n=1}^{\infty} \frac{2}{L\kappa_n} \sin(\kappa_n L) \frac{e^{-\sqrt{\kappa_n^2+1/4}X} - e^{\sqrt{\kappa_n^2+1/4}(2l+X)}}{1 - e^{2l\sqrt{\kappa_n^2+1/4}}} \cos \kappa_n Y \quad (29)$$

From Eq. (26) the triple flame temperature is obtained as:

$$\Theta = \sum_{\kappa_n=1}^{\infty} e^{\frac{1}{2}X} \frac{2}{L\kappa_n} \sin(\kappa_n L) \left(\frac{e^{-\sqrt{\kappa_n^2+1/4}X} - e^{\sqrt{\kappa_n^2+1/4}(2l+X)}}{1 - e^{2l\sqrt{\kappa_n^2+1/4}}} \right) \cos \kappa_n Y \quad (30)$$

$$\text{where } \kappa_n = \frac{(2n-1)\pi}{2L}$$

FORMULATIONS WITH VAPORIZATION

The problem presented here will undergo the same method as presented in the previous section; however, here the fuel stream consists of uniformly distributed volatile fuel droplets. It is presumed that the fuel droplets vaporize to form a known gaseous compound which is then oxidized and the products are assumed to be in the gaseous phase.

Basic equations with vaporization

The kinetic of vaporization are presumed to be represented by the expression:¹⁵

$$W_v = Bn_s(4\pi r^2)T^n \quad (31)$$

where W_v is the mass of gaseous fuel vaporized per unit volume per second. The quantities B and n are constants, which are presumed to be known, and T denotes the gas temperature. For simplicity, it is assumed that the droplet temperature is approximately equal to the gas temperature.

The time evolution of the temperature, T , is given by:

$$\rho c_p \frac{DT}{Dt} = D_T \nabla^2 T + Q\omega - W_v Q_v \quad (32)$$

where W_v is vaporization term described by Eq. (31) and Q_v is the heat associated with a vaporizing unit mass of the fuel. With the Lewis number unity, Eq. (32) becomes:

$$\rho c_p \frac{DT}{Dt} = k\rho_{\infty} c_p \nabla^2 T + Q\omega - W_v Q_v \quad (33)$$

Dimensionless system of governing equations

Following the same approach as in the preceding section, Eq. (32) may be written in terms of dimensionless variables as:

$$\bar{\omega}(U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y}) = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} + \lambda \bar{\omega}^{v_f + v_x} \Sigma(Z/Z_s, \Theta) \exp\left[-\frac{\beta(1-\Theta)}{1-\alpha(1-\Theta)}\right] - \gamma \Theta^n Q_v \quad (34)$$

where γ is defined by:

$$\gamma = \frac{Bn_s(4\pi r^2)}{c_p} \left(\frac{QZ_s}{m_F V_{FCp}}\right)^{n-1} \quad (35)$$

Activation energy asymptotic

Here, as in the previous section, $\bar{\omega}$ is introduced and taken to be unity. Two cases are studied, the first $U = 1$ and $V = 0$ and the second $V = 1$, $U = 1$, in order to study the effects the flow velocity components on the flame temperature field.

Case 1, $U = 1$ and $V = 0$: Eq. (34) becomes:

$$\frac{\partial \Theta}{\partial X} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} + \lambda \Sigma(Z/Z_s, \Theta) \exp[-\beta(1-\Theta)] - \gamma \Theta^n Q_v \quad (36)$$

and with the AEA assumption, one obtains:

$$\frac{\partial \Theta}{\partial X} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} - \gamma^* \Theta \quad (37)$$

where $\gamma^* = \gamma Q_v$. Equation (37) is a linear homogeneous partial differential equation which is converted to a Helmholtz differential equation by changing the variable Θ to ψ by the same substitution, $\Theta(X, Y) = \exp(1/2X)\psi(X, Y)$.

Hence, Eq. (37) becomes:

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} - (\gamma^* + \frac{1}{4})\psi = 0 \quad (38)$$

Equation (38) is a two-dimensional homogeneous Helmholtz equation in the rectangular Cartesian system of coordinates, which may be solved by the separation of variables method. The boundary conditions are also similar to that of the case without vaporization:

$$\psi = \sum_n \frac{2 \sin(\kappa_n L) \exp\left(-\sqrt{\kappa_n^2 + (\gamma^* + \frac{1}{4})} X\right) - \exp\left(\sqrt{\kappa_n^2 + (\gamma^* + \frac{1}{4})} (2l + X)\right)}{L \kappa_n \left(1 - \exp\left(2l \sqrt{\kappa_n^2 + (\gamma^* + \frac{1}{4})}\right)\right)} \cos \kappa_n Y \quad (39)$$

Therefore, the triple flame temperature is obtained:

$$\Theta = \sum_n \exp\left(\frac{1}{2} X\right) \frac{2 \sin(\kappa_n L)}{L \kappa_n} \times \frac{\exp\left(-\sqrt{\kappa_n^2 + (\gamma^* + \frac{1}{4})} X\right) - \exp\left(\sqrt{\kappa_n^2 + (\gamma^* + \frac{1}{4})} (2l + X)\right)}{1 - \exp\left(2l \sqrt{\kappa_n^2 + (\gamma^* + \frac{1}{4})}\right)} \cos \kappa_n Y \quad (40)$$

Case 2. $V = 1, U = 1$: Eq. Eq. (34) becomes:

$$\frac{\partial \Theta}{\partial X} + \frac{\partial \Theta}{\partial Y} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} + \lambda \Sigma (Z / Z_s, \Theta) \exp[-\beta(1 - \Theta)] - \gamma \Theta^n Q_v \quad (41)$$

Considering large activation energy, Eq. (41) becomes:

$$\frac{\partial \Theta}{\partial X} + \frac{\partial \Theta}{\partial Y} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} - \gamma^* \Theta \quad (42)$$

Equation (42) is also a linear homogeneous partial differential equation, which is converted to Helmholtz differential equation by changing Θ to ψ by the substitution $\Theta(X, Y) = \exp(1/2(X + Y))\psi(X, Y)$, giving:

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} - (\gamma^* + \frac{1}{2})\psi = 0 \quad (43)$$

Equation (43) is also a two-dimensional homogeneous Helmholtz equation in the rectangular Cartesian system of coordinates. However the boundary conditions here would become:

$$\begin{aligned} \psi(-l, Y) &= 0 \\ \psi(0, Y) &= \exp(-Y/2) \\ \psi_Y(X, 0) &= 0 \\ \psi(X, -L) &= 0 \end{aligned}$$

Therefore, one obtains:

$$\psi = \sum_{n=1}^{\infty} \left(\frac{\exp(-\frac{L}{2})\sin(\kappa_n L) - \frac{3\exp(-\frac{L}{2})}{4\kappa_n^2} \cos(\kappa_n L) + \frac{3}{4\kappa_n^2}}{\frac{L}{2} + \frac{\sin(\kappa_n L)}{4\kappa_n}} \right) \times \frac{\exp(-(\sqrt{\kappa_n^2 + \gamma^* + \frac{1}{2}})X) - \exp((\sqrt{\kappa_n^2 + \gamma^* + \frac{1}{2}})(2l + X))}{1 - \exp(2l\sqrt{\kappa_n^2 + \gamma^* + \frac{1}{2}})} \cos \kappa_n Y \quad (44)$$

Thus, Θ the triple flame temperature is obtained:

$$\Theta = \sum_{n=1}^{\infty} \exp\left(\frac{1}{2}(X + Y)\right) \left(\frac{\exp(-\frac{L}{2})\sin(\kappa_n L) - \frac{3\exp(-\frac{L}{2})}{4\kappa_n^2} \cos(\kappa_n L) + \frac{3}{4\kappa_n^2}}{\frac{L}{2} + \frac{\sin(\kappa_n L)}{4\kappa_n}} \right) \times \frac{\exp(-(\sqrt{\kappa_n^2 + \gamma^* + \frac{1}{2}})X) - \exp((\sqrt{\kappa_n^2 + \gamma^* + \frac{1}{2}})(2l + X))}{1 - \exp(2l\sqrt{\kappa_n^2 + \gamma^* + \frac{1}{2}})} \cos \kappa_n Y \quad (45)$$

RESULTS AND DISCUSSION

Some results may now be presented for variations of the dimensionless flame temperature *versus* the dimensionless coordinates. The three-dimensional diagram of the flame temperature for a non-vaporizing fuel stream *versus* the coordinates X and Y is shown in Fig. 2. The results of the flame contour are compared to experimental results,¹¹ which are displayed in Fig. 3.

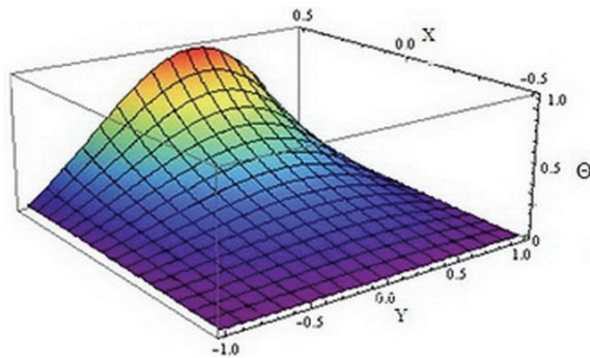


Fig. 2. Three-dimensional diagram of dimensionless flame temperature for non-vaporizing fuel stream *vs.* the dimensionless coordinates X and Y .

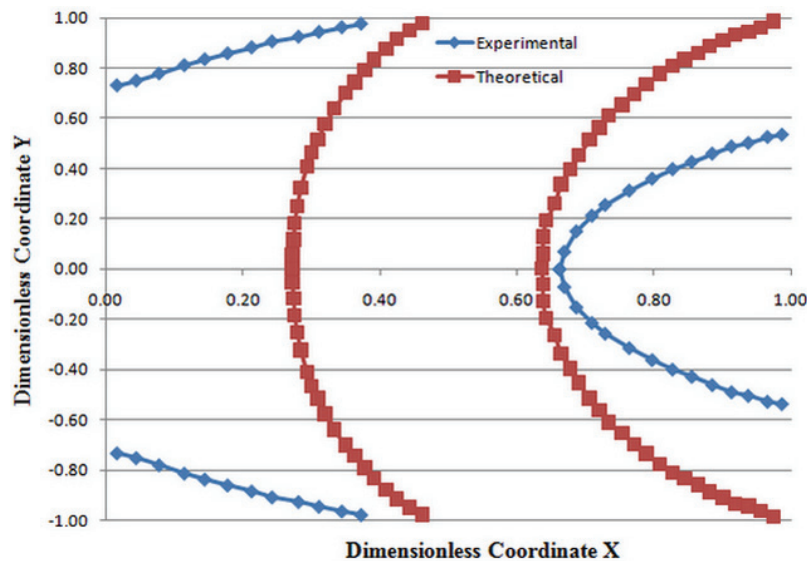


Fig. 3. Comparison of the theoretical and experimental results of the flame contour.

As can be observed in Fig. 3, the trend of flame contour for the theoretical solution presented in this paper is in good agreement with the experiments. In order to simplify the solution of the current governing differential equations, it was assumed that the location of the flame formation was at the origin of the Cartesian coordinate axis; this assumption is applied to the equation by the boun-

dary conditions. While in reality the flame would be formed after mixing the fuel and oxidizer. Therefore the location of flame formation would be slightly shifted and displaced through the reactant stream propagation, namely in the positive X -direction (see Fig. 1b). As a result, referring to Fig. 3, it could be observed that for small values of X (very close to the Y -axis), the discrepancy between the theoretical and experimental results is quite significant, while it should be noticed that in spite of the discrepancy at the values of X close to unity due to the effects of the assumptions, in far upstream, the experimental results show more agreement with the theoretical results. Since in reality combustion is an unsteady phenomenon, flame propagation would be decreased in the places far from the stoichiometric line, while the steady state condition was studied in the presented work.

The variation of temperature along the X -axis is demonstrated in Fig. 4 for both sections of the non-vaporizing fuel stream and the volatile droplet stream. In this diagram, it may be seen that the temperature increases moving along the X -axis. In the case in which the fuel droplets vaporized into the gaseous phase, the curve of the temperature is located lower than in the case of the non-vaporizing fuel stream, because vaporization heat is given to the fuel droplets in order to convert them into the vapor phase. Therefore, the temperature has a lower value than the case of the non-vaporizing fuel. Moreover, the stream temperature with vaporization of the fuel droplets was studied under two fuel flow conditions; the former diagram (case 1) in which only the velocity parallel to the reactant flow

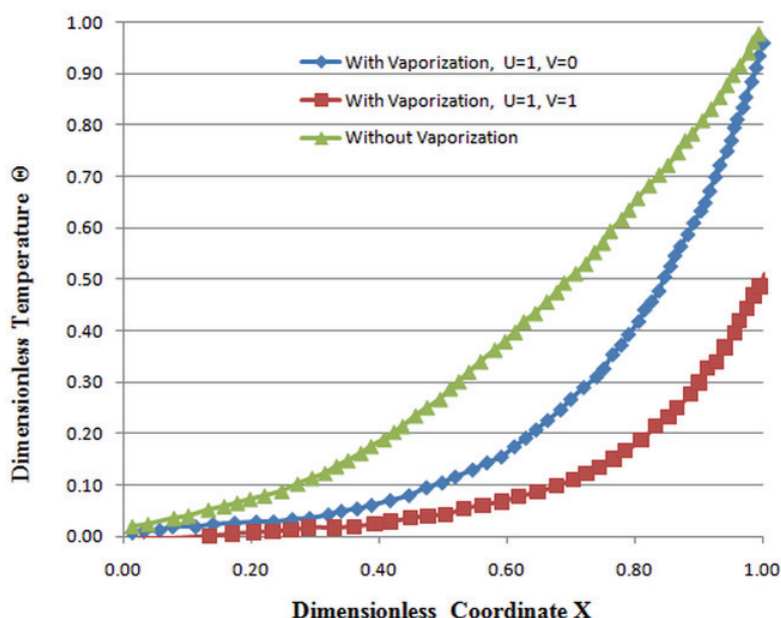


Fig. 4. Flame temperature vs. X -axis, comparison between non-vaporizing and volatile fuel streams; as well as the effects of the vertical velocity component of fuel flow.

(U) is considered with the vertical velocity being neglected and the latter one (case 2) in which the vertical velocity (V) was also considered. Therefore the effects of this additional term were studied. From Fig. 4, it can be seen that the inclusion of the vertical velocity in the energy equation affected a further shift to the right of the flame temperature along the X -axis.

The temperature variation along the Y -axis is illustrated in Fig. 5 in the same way as was the case for the X -axis. Here also, the effect of the additional term for the vertical velocity of the fuel stream was studied. As can be seen from Fig. 5, the flame temperature had its peak value at $Y = 0$, where the fuel and oxidizer were mixed in stoichiometric amounts, while when considering the velocity perpendicular to the flow path, the flame temperature peak value moved to the right and downwards. Thus, the extra momentum which is brought into the flow by considering the velocity perpendicular to the flow path causes a decrease in the maximum flame temperature and the location of the temperature peak is moved to the right.

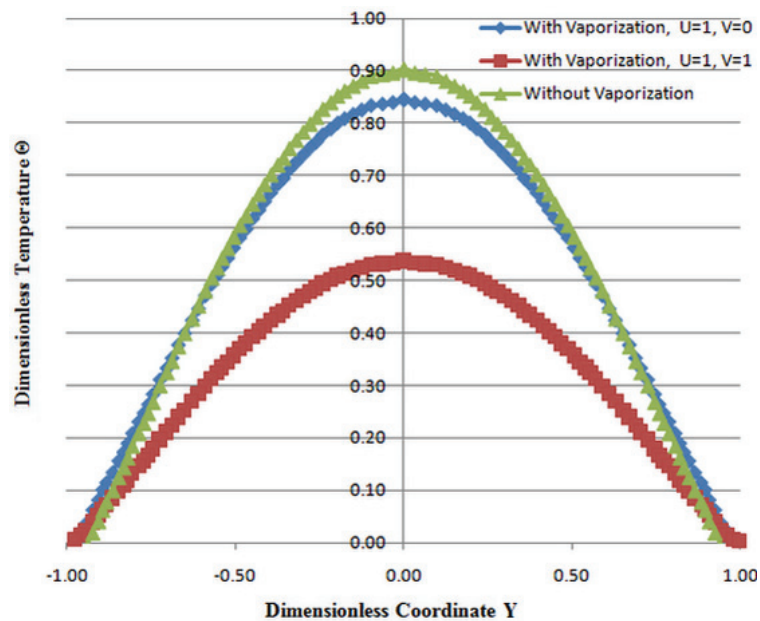


Fig. 5. Flame temperature vs. Y -axis, comparison between non-vaporizing and volatile fuel streams; as well as the effects of the vertical velocity component of fuel flow.

The flame temperature variations along the X - and Y -axis for particles of various radii are illustrated in Figs. 6 and 7, respectively. It can be observed that increasing the particle radius results in a decrease in the flame temperature. The smaller the particle, the higher would be the flame temperature due to the lower burning surface.

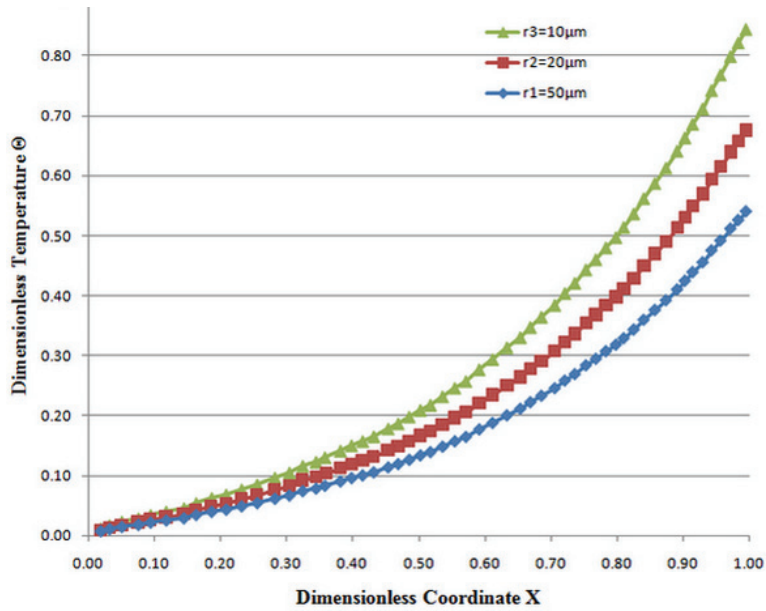


Fig. 6. Flame temperature vs. X-axis for various particle radii.

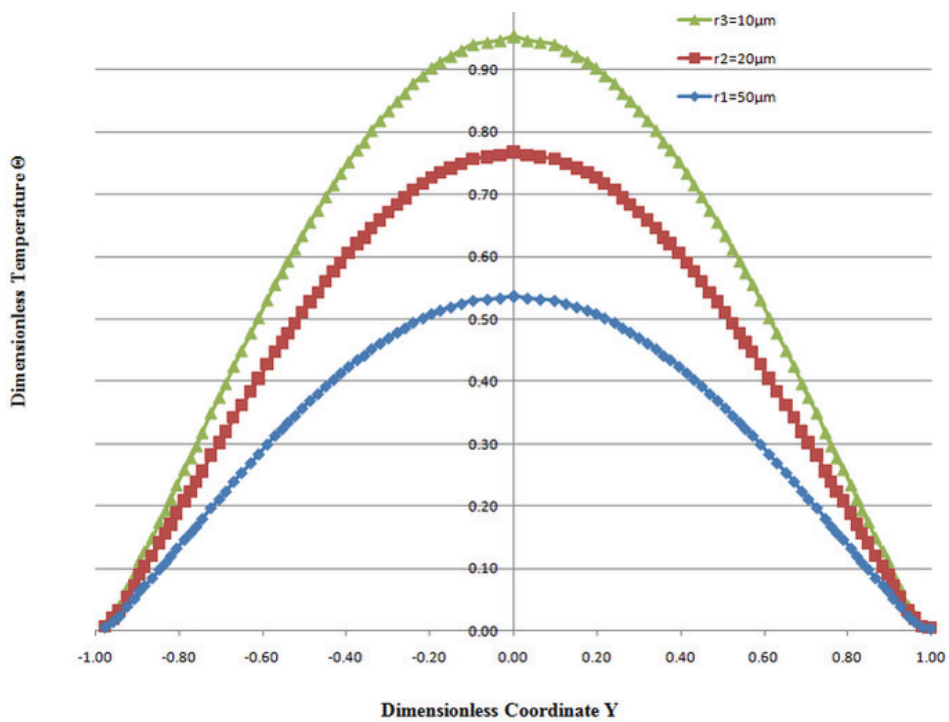


Fig. 7. Flame temperature vs. Y-axis for various particle radii.

The results shown are applicable to any triple flame configuration. According to the assumptions used in the present article, any volatile fuel particle could be considered in this study. The overall flame configurations are the same, because the methods within which the fuel and oxidizer are mixed and as well the governing equations are the same for different fuel droplets. The differences are related to some constant parameters in the equations, such as fuel density, the mixture fraction of the reactants, *etc.* Nevertheless the total schematic of the flame as shown in Fig. 1 is still valid. Therefore the presented results are not only applicable to a specific flame, but can also be applied to any triple flame configuration.

CONCLUSIONS

In this article, an analytical approach was developed for assessing the flame structure and combustion properties of a triple flame containing uniformly distributed volatile fuel droplet. The obtained results were compared to the results obtained for non-vaporizing fuel particles. Furthermore, the role of vaporization term was investigated in this research for two different approaches in which the stream velocity along the flow and perpendicular to the direction of the flow were considered. The calculated results were compared to the experimental data. When the vaporization term for the fuel droplets was included, it was observed that the temperature field of the triple flame was reduced compared to that for a non-vaporizing fuel stream because of the heat removed from the flow. In addition, by studying the temperature field with flow velocity effects along horizontal and vertical direction, it was demonstrated that the flame temperature was decreased due to the extra momentum brought into the flow comparing to the case in which only the horizontal velocity component was considered.

The calculation method used in this article is based on an analytical approach in which all the governing equations were derived according to the related energy equation in terms of temperature and the equations were written in the non-dimensional form by using dimensionless variables. In the analytical approach, no attention was paid to the analysis of the triple flame. However, experimental results were compared with the results of this study, and the terms of agreements and discrepancies are described in the discussion. Hence, the approach employed to follow the steps of combustion within a triple flame is not a numerical method; however, there are many such methods, e.g., discretization of the equation within a finely meshed domain, which may be used to verify the results.

ИЗВОД

АНАЛИТИЧКО ИСПИТИВАЊЕ ЕФЕКТА ИСПАРЉИВОСТИ НА ТРОСТРУКИ
ПЛАМЕН СА ДВОДИМЕНЗИОНИМ КАПЉИЦАМА

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Испитивана је структура троструког пламена који пропагира у системима са сагоревањем и који садржи дистрибуиране честице испарљивог горива. Анализа је извршена за једностепену иреверзибилну реакцију са асимптотском границом, где је вредност Zeldovich-овог броја велика. Коришћењем јединичног Lewis-ог броја, постављен је математички модел и добијено је аналитичко решење за температуре у троструком пламену, Анализа је урађена у два дела. У првом делу је третирана неиспарљива струја горива, док је у другом делу узета у обзир испарљивост капљица горива. Претпостављено је да капљице горива испаравају и доприносе гасовитом гориву познате хемијске структуре, које се затим оксидише у гасовитој фази. Овде су разматрана два случаја. У првом случају, узима се у обзир само брзина паралелна са током реактанта, док се у другом случају разматра и вертикална брзина. Једначине енергетског биланса су решене и представљене су једначине температурног поља. Прво су приказани резултати за неиспарљиво гориво и упоређени са експериментима. Резултати температурног поља за испарљиво гориво су такође приказани у оквиру поређења горе поменутих случајева, што показује ефекте разматрања вертикалне компоненте брзине на температурно поље.

(Примљено 13. августа 2010, ревидирано 10. јануара 2011)

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